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Hence, assuming $x_1^2+y_1^2=x_2^2$ and $x_1^2-y_1^2=y_2^2$
we have shown that if (3) is satisfied by x_1, y_1, z_1, then it follows that there is a relation $b^4-k^4=\square$ where $b<x_1$. Using the second assumption, and the only other possible one, namely,

$$\begin{cases} x_1^2+y_1^2=2x_2^2 \\ x_1^2-y_1^2=2y_2^2 \end{cases}$$

we obtain, by addition and subtraction,

$$x_2^2+y_2^2=x_1^2\ldots\ldots(9),$$
$$x_2^2-y_2^2=y_1^2\ldots\ldots(10),$$

and multiplying, $x_2^4-y_2^4=\square$. But (9) gives $x_2<x_1$.

Hence it has been shown that using this assumption, we can obtain from (3) a relation $x_2^4-y_2^4=\square$ such that $x_2<x_1$. Therefore, if (3) holds it is *always* possible to find a relation $x_2^4-y_2^4=z_2^2$ such that $x_2<x_1$.

From $x_2^4-y_2^4=z_2^2$ it is then possible to find $x_3^4-y_3^4=z_3^2$ where $x_3<x_2$, and so we can get any number of equations, of the type, $x_n^4-y_n^4=z_n^2$, where

$$x_n<_{n-1}<x_{n-2}\ldots\ldots<x_3<x_2<x_1.$$

Since x_n is always positive, by taking n sufficiently large, there is obtained $1-y_n^4=z_n^2$, which is impossible for positive integral values of y_n and z_n.

Hence the impossibility of $x_1^4-y_1^4=z_1^2$ has been completely established, and therefore the fact that $xy(x^2-y^2)=z^2$ cannot be satisfied, follows.

105. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

If the refractive index of a medium at any point be $\mu=x$, prove that the path of the ray will be the curve $\dfrac{2x}{a}=\dfrac{c}{a}\epsilon^{y/a}+\dfrac{a}{c}\epsilon^{-(y/a)}$, a and c being constants.

Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy. Ohio University, Athens, O.

Let (x, y) be any point in the path; $\mu=kx$, the index of refraction; and $p=dy/dx$. The differential equation to the path is

$$\frac{dp/dx}{1+p^2}=\frac{1}{\mu}\left(\frac{d\mu}{dy}-\frac{d\mu}{dx}\frac{dy}{dx}\right)\ldots\ldots(1).$$

$d\mu/dx=k$, and (1) reduces to

$$\frac{dp}{p(1+p^2)}=-\frac{dx}{x}\ldots\ldots(2).$$

Integrating, $\log\frac{px}{\sqrt{(1+p^2)}}=C=\log a$, say......(3).

This gives $p=\frac{dy}{dx}=\frac{a}{\sqrt{(x^2-a^2)}}$......(4).

Integrating (4), $y=a\log[x+\sqrt{(x^2-a^2)}]+C'$......(5).

Let $y=0$, when $x=b$; then $C'=-a\log[b+\sqrt{(b^2-a^2)}]$, and (5) becomes after putting $c=b+\sqrt{(b^2-a^2)}$,

$$\frac{2x}{a}=\frac{c}{a}e^{y/a}+\frac{a}{c}e^{-(y/a)}......(6),$$

e being the Naperian base.

Also solved by *G. B. M. ZERR* and *L. C. WALKER*.

106. Proposed by **J. W. YOUNG**, Oliver Graduate Student in Mathematics, Cornell University, Ithaca, N. Y.

Prove that $\frac{(2m)!}{(m!)^2}$ is an integer; and more generally that $\frac{(nm)!}{(m!)^n}$ is an integer; m, n being any positive integers.

Solution by **G. B. M. ZERR**, A. M., Ph. D., The Temple College, Philadelphia, Pa.; and **L. C. WALKER**, A. M., Professor of Mathematics, Petaluma High School, Petaluma, Cal.

It has been demonstrated that the product of any n successive integers is divisible by $n!$

$$\frac{(nm)!}{(m!)^n}\div\frac{[(n-1)m]!}{(m!)^{n-1}}=\frac{(nm)!}{[(n-1)m)!}\cdot\frac{1}{m!}=\frac{(nm)!}{(nm-m)!}\cdot\frac{1}{m!}$$

$$=\frac{nm(nm-1)(nm-2)......\text{to } m \text{ factors}}{m!}=\text{an integer}.$$

$\therefore$ If $\frac{[(n-1)m]!}{(m!)^{n-1}}$ is an integer, so is $\frac{(nm)!}{(m!)^n}$.

But $\frac{m!}{m!}$ is an integer. $\therefore$ $\frac{(2m)!}{(m!)^2}$ is an integer, and so on to $\frac{(nm)!}{(m!)^n}$.

Also solved by *H. S. VANDIVER*.